Exercise 26

Use a calculator to evaluate the line integral correct to four decimal places.

$$\int_C z\ln(x+y)\,ds,\quad \text{where } C \text{ has parametric equations } x = 1+3t,\ y = 2+t^2,\ z = t^4,\quad -1 \le t \le 1$$

Solution

With the given parameterization in t, the line integral becomes

$$\begin{aligned}\int_C z\ln(x+y)\,ds &= \int_{-1}^{1} z(t)\ln[x(t)+y(t)]\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}\,dt \\ &= \int_{-1}^{1} (t^4)\ln[(1+3t)+(2+t^2)]\sqrt{(3)^2+(2t)^2+(4t^3)^2}\,dt \\ &= \int_{-1}^{1} t^4\ln(3+3t+t^2)\sqrt{9+4t^2+16t^6}\,dt.\end{aligned}$$

Let $f(t)$ represent the integrand.

$$\int_C z\ln(x+y)\,ds = \int_{-1}^{1} f(t)\,dt$$

Use Simpson's rule with $n = 10$.

$$\begin{aligned}\int_C z\ln(x+y)\,ds &\approx \frac{\Delta t}{3}[f(t_0)+4f(t_1)+2f(t_2)+4f(t_3)+2f(t_4)+4f(t_5) \\ &\qquad + 2f(t_6)+4f(t_7)+2f(t_8)+4f(t_9)+f(t_{10})] \\ &\approx \frac{1-(-1)}{3(10)}\left[f(-1)+4f\left(-\frac{4}{5}\right)+2f\left(-\frac{3}{5}\right)+4f\left(-\frac{2}{5}\right)+2f\left(-\frac{1}{5}\right)+4f(0)\right. \\ &\qquad \left.+2f\left(\frac{1}{5}\right)+4f\left(\frac{2}{5}\right)+2f\left(\frac{3}{5}\right)+4f\left(\frac{4}{5}\right)+f(1)\right] \\ &\approx \frac{1-(-1)}{3(10)}(26.0868) \\ &\approx 1.73912\end{aligned}$$